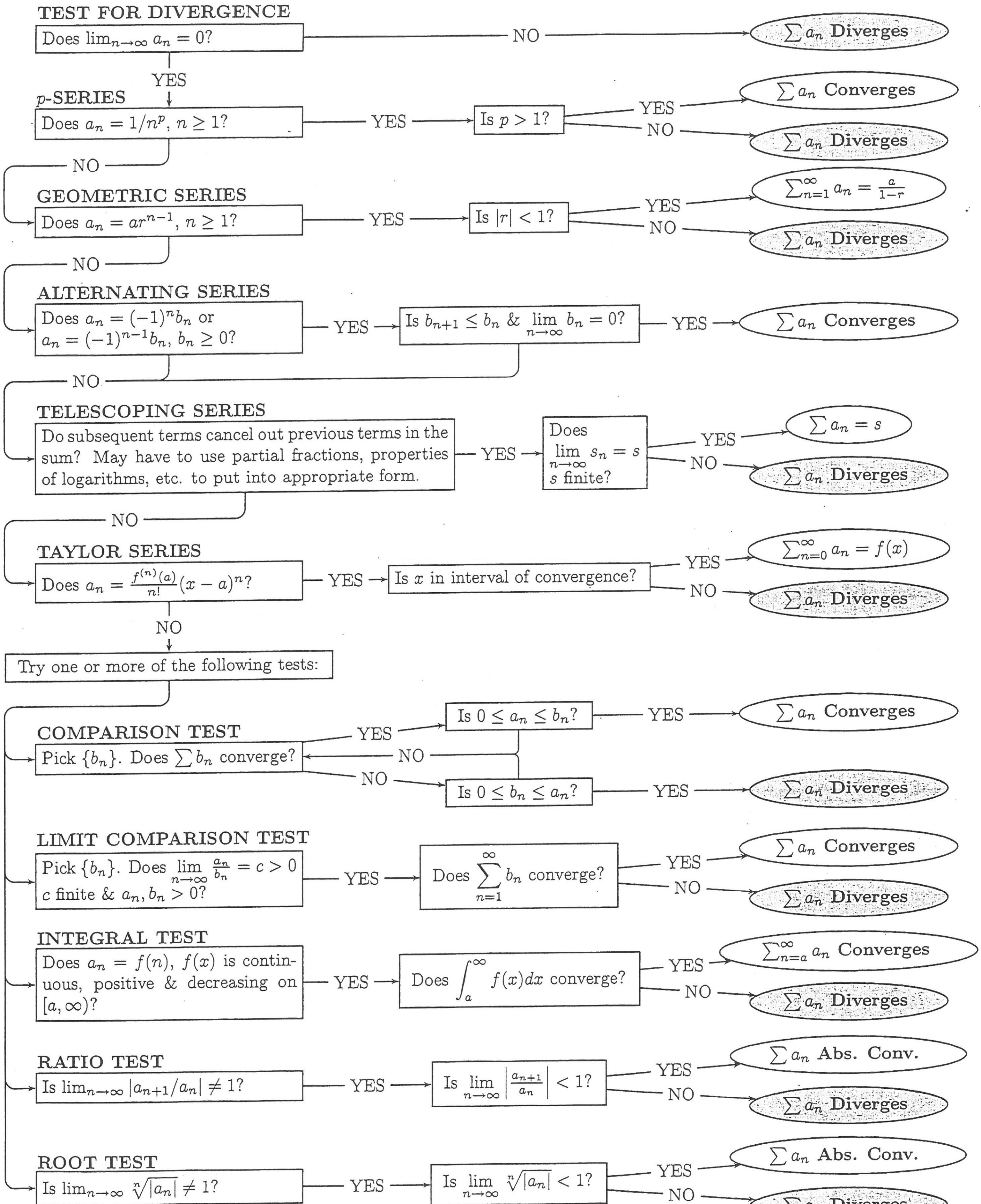
SERIES CONVERGENCE/DIVERGENCE FLOW CHART



Summary of Convergence Tests for Series

Test	Series	Convergence or Divergence	Comments
n^{th} term test (or the zero test)	$\sum a_n$	Diverges if $\lim_{n\to\infty} a_n \neq 0$	Inconclusive if $\lim_{n\to\infty} a_n = 0$.
Geometric series	$\sum_{n=0}^{\infty} ar^n \left(\text{or} \sum_{n=1}^{\infty} ar^{n-1}\right)$	Converges to $\frac{a}{1-x}$ only if $\|x\| < 1$ Diverges if $\|x\| \geq 1$	Useful for comparison tests if the n^{th} term a_n of a series is similar to ax^n.
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$ Diverges if $p \leq 1$	Useful for comparison tests if the n^{th} term a_n of a series is similar to $\frac{1}{n^p}$.
Integral	$\sum_{n=c}^{\infty} a_n \quad (c \geq 0)$ $a_n = f(n)$ for all n	Converges if $\int_c^{\infty} f(x)\,dx$ converges Diverges if $\int_c^{\infty} f(x)\,dx$ diverges	The function f obtained from $a_n = f(n)$ must be continuous, positive, decreasing and readily integrable for $x \geq c$.
Comparison	$\sum a_n$ and $\sum b_n$ with $0 \leq a_n \leq b_n$ for all n	$\sum b_n$ converges $\Longrightarrow \sum a_n$ converges $\sum a_n$ diverges $\Longrightarrow \sum b_n$ diverges	The comparison series $\sum b_n$ is often a geometric series or a p-series.
Limit Comparison*	$\sum a_n$ and $\sum b_n$ with $a_n, b_n > 0$ for all n and $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$	$\sum b_n$ converges $\Longrightarrow \sum a_n$ converges $\sum b_n$ diverges $\Longrightarrow \sum a_n$ diverges	The comparison series $\sum b_n$ is often a geometric series or a p-series. To find b_n consider only the terms of a_n that have the greatest effect on the magnitude.
Ratio	$\sum a_n$ with $\lim_{n\to\infty} \frac{\|a_{n+1}\|}{\|a_n\|} = L$	Converges (absolutely) if $L < 1$ Diverges if $L > 1$ or if L is infinite	Inconclusive if $L = 1$. Useful if a_n involves factorials or n^{th} powers.
Root*	$\sum a_n$ with $\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = L$	Converges (absolutely) if $L < 1$ Diverges if $L > 1$ or if L is infinite	Test is inconclusive if $L = 1$. Useful if a_n involves n^{th} powers.
Absolute Value $\sum \|a_n\|$	$\sum a_n$	$\sum \|a_n\|$ converges $\Longrightarrow \sum a_n$ converges	Useful for series containing both positive and negative terms.
Alternating series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n \quad (a_n > 0)$	Converges if $0 < a_{n+1} < a_n$ for all n and $\lim_{n\to\infty} a_n = 0$	Applicable only to series with alternating terms.

*The Root and Limit Comparison tests are not included in the current textbook used in Calculus classes at Bates College.